M theory and the ''integrating in'' method with an antisymmetric tensor

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Recently, a nonhyperelliptic curve describing the Coulomb branch of $N=2$ SUSY SU(N_c) Yang-Mills theory with antisymmetric tensor matter was proposed using the configuration of a single M theory five-brane. We study the singular surface in the moduli space of the curve to compare it with results from the ''integrating in'' method in field theory. In order to achieve consistency, we find it necessary to take into account an additional superpotential W_Δ which has been neglected so far. The explicit form of W_Δ is worked out. [S0556-2821(98)02916-6]

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I. INTRODUCTION

In recent years, a deeper understanding of supersymmetric (SUSY) gauge theories in various dimensions has been gained by realizing them on the world volumes of D-branes [1–22]. Witten [23] also pointed out that intersecting brane configurations of type IIA string theory corresponding to $N=2$ SUSY gauge theories in four dimensions can be described by a single M theory five-brane wrapping around a Riemann surface. The Riemann surface is nothing but the Seiberg-Witten curve [24] and therefore the five-brane configuration contains the structure of the moduli space of vacua. The M-theoretic method is also applied to discuss various aspects of SUSY gauge theories [25–41] and found to be quite useful to understand them.

On the other hand, field-theoretic approaches also provide us with important information on the Seiberg-Witten curves. One of them is based on the deformation to $N=1$ SUSY. The moduli space of the $N=2$ SUSY vacua in the Coulomb phase exhibits singularities where solitons such as monopoles or dyons become massless. When $N=2$ SUSY gauge theories are broken to $N=1$ SUSY by perturbations of tree-level superpotentials, only these singularities remain as $N=1$ SUSY vacua [24]. Conversely, we can tune parameters of superpotentials in $N=1$ SUSY Yang-Mills theories with an adjoint matter field in order to obtain $N=2$ SUSY Yang-Mills theories. By this procedure, one expects that the singularity surfaces in $N=2$ moduli space can be reached. Thus, by studying the low energy effective action of $N=1$ Yang-Mills theory with an adjoint matter field with a tree-level superpotential chosen properly, we can derive some information on the singular surface of the $N=2$ moduli spaces. In fact, Elitzur *et al.* have developed a method to obtain the singularity surfaces in $N=2$ SUSY Yang-Mills theories by using a single confined photon in $N=1$ SUSY gauge theories [42]. In this way, the curve of the $N=2$ SUSY theory can be recovered by ''integrating in'' [43] the adjoint matter fields in the $N=1$ low energy effective theory. This ''integrating in'' method has been extended to SUSY Yang-Mills theories with various gauge groups including exceptional groups [44–48]. In general, however, the effective superpotential is not completely fixed by symmetries and holomorphy. Possible additional terms are usually denoted as W_Δ. In these ''integrating in'' approaches, a crucial assumption has been made: the low-energy effective superpotential has a minimal form, namely, $W_\Delta=0$. So far, this has provided us with consistent results.

Recently, Landsteiner and Lopez [32] have proposed a nonhyperelliptic curve describing the Coulomb branch of $N=2$ SUSY SU(N_c) Yang-Mills theory with antisymmetric tensor matter from a configuration of a single M theory five-brane. Although the proposed curve passes some consistency checks, it seems necessary to make sure of it further from other points of view. The purpose of this paper is to obtain the singularity surface of $N=2$ SUSY SU(N_c) Yang-Mills theory with antisymmetric tensor matter by using the ''integrating in'' method. We find that the usual ''integrating in'' method assuming $W_\Delta=0$ gives a singularity surface which disagrees with the proposed M-theoretic curve. By assuming that the brane configuration is correct, we find that there exists a nontrivial $W_\Delta\neq 0$ which gives a singular surface consistent with the M-theoretic curve at least up to certain high powers of the dynamical scale Λ of the gauge interactions. Our results can be regarded as evidence for the necessity of nontrivial W_Δ.

Section II gives a brief review of the brane configuration in M theory describing $N=2$ SUSY SU(N_c) Yang-Mills theory with antisymmetric tensor matter. In Sec. III, we discuss the singular surface of the moduli space assuming $W_\Delta=0$. It is shown that the singular surface is inconsistent with the M-theoretic curve obtained from the brane configuration. In Sec. IV, we derive the explicit form of $W_\Delta\neq 0$ by requiring consistency of the singular surface with the M-theoretic curve. Section V contains a discussion.

II. BRANE CONFIGURATION

In this section we briefly review the brane configuration in M theory describing the Coulomb branch of $N=2$ SUSY SU(N_c) Yang-Mills theory with antisymmetric tensor matter [32]. Let us first examine the brane configuration in the type

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IIA string picture. Consider type IIA string theory in flat space-time where x^0 denotes the time coordinate and $x^1, \ldots, x^9$ denote the space coordinates. The brane configuration consists of an orientifold six-plane of charge -4 with the world-volume coordinates $(x^0,x^1,x^2,x^3,x^7,x^8,x^9)$, Neveu-Schwarz (NS) 5-branes with the world-volume coordinates $(x^0,x^1,x^2,x^3,x^4,x^5)$, and Dirichlet (D) 4-branes with the world-volume coordinates (x^0,x^1,x^2,x^3,x^6). The orientifold six-plane sits at $x^4=x^5=x^6=0$. This means that the space-time should be identified under the transformation

$$(x^4,x^5,x^6)\rightarrow(-x^4,-x^5,-x^6). \tag{2.1}$$

One NS5-brane is placed on top of the orientifold six-plane and the other NS5-brane is to the right of it. Further there are N_c D4-branes stretching in between the NS5-branes. In the left of the orientifold six-plane we have of course the mirror image of these branes. The D4-branes have a finite extent in the x^6 direction. The four-dimensional $N=2$ SUSY gauge theory we discuss is defined on the world-volume coordinates (x^0,x^1,x^2,x^3) of the D4-branes. When all N_c D4-branes coincide, the open strings connecting N_c D4-branes in the left of the orientifold six-plane give the SU(N_c) gauge vector multiplets. The open strings connecting the left and right D4-branes give a hypermultiplet in the antisymmetric representation of the gauge group because of the presence of the orientifold six-plane.

The brane configuration can be reinterpreted in M theory as a configuration of a single five-brane embedded in the 11-dimensional space-time $\mathbb{R}^7\times S$ where S is the Atiyah-Hitchin space [49,50]. The $\mathbb{R}^7$ spans the 0123789 directions, while S spans the 456 directions in the type IIA limit and wraps around the circle in the 11th direction x^{10} whose radius is denoted by R. The five-brane world volume becomes $\mathbb{R}^4\times\Sigma$ where $\mathbb{R}^4$ spans the 0123 directions while Σ is a curve embedded in the Atiyah-Hitchin space S whose complex structure is represented as $xy=\Lambda^{2N_c+4}v^{-4}$, where $v=x^4+ix^5$. For large y with x fixed, y tends to $t=\exp[-(x^6+ix^{10})/R]$, while for large x with y fixed we have $x\sim t^{-1}$. In the M-theoretic brane configuration, Λ represents the mass scale corresponding to the dynamical scale of gauge interaction in field theory. The curve Σ is not hyperelliptic, contrary to the case of the $N=2$ SUSY QCD where matter hypermultiplets are only in the fundamental representations.

Since there are three NS5-branes involved, the M-theoretic curve describing the brane configuration becomes cubic in y. By using symmetry under $x\leftrightarrow y$, $v\leftrightarrow -v$ and other arguments, Landsteiner and Lopez have found the following curve Σ for the above brane configuration in M theory, and proposed it to describe $N=2$ SUSY SU(N_c) Yang-Mills gauge theory with an antisymmetric tensor matter field [32]:

$$y^3+y^2[p(v)+3\Lambda^{N_c+2}v^{-2}] \\ +y\Lambda^{N_c+2}v^{-2}[q(v)+3\Lambda^{N_c+2}v^{-2}]+\Lambda^{3N_c+6}v^{-6}=0, \tag{2.2}$$

where

$$p(v)=\prod_{i=1}^{N_c}(v-a_i) \quad \text{and} \quad q(v)=p(-v). \tag{2.3}$$

The N_c parameters a_i represent the positions of the D4-branes in the type IIA string picture.

We denote by Φ an $N=1$ chiral superfield in the adjoint representation in the SU(N_c) gauge group. Together with the $N=1$ vector multiplet V in the adjoint representation, it forms an $N=2$ vector multiplet. In addition to them, we have antisymmetric tensor matter A^{ij} and its conjugate $\tilde{A}_{ij}$, where $i,j=1,2,\ldots,N_c$ are color indices. Both of them are the $N=1$ chiral superfields and form together an $N=2$ hypermultiplet. The tree-level superpotential $W_{\rm tree}$ contains a tree-level mass parameter m of the antisymmetric tensor matter:

$$W_{\rm tree}=\sqrt{2}\sum_{\substack{i<j\\k<l}}\tilde{A}_{ij}(\Phi^i{}_k\delta^j{}_l+\delta^i{}_k\Phi^j{}_l+m\,\delta^i{}_k\delta^j{}_l)A^{kl}. \tag{2.4}$$

The distance between the average position of the D4-branes on the left and the average position of the D4-branes on the right is equal to the tree-level mass parameter m of antisymmetric tensor matter:

$$m=\frac{2}{N_c}\sum_{i=1}^{N_c}a_i. \tag{2.5}$$

The distance between the position of each D4-brane and the average position of the D4-branes on the left corresponds to the vacuum expectation value (VEV) of the diagonal element ϕ_i of the adjoint matter Φ,

$$a_i=\frac{m}{2}+\langle\phi_i\rangle \quad (i=1,\ldots,N_c), \tag{2.6}$$

where the VEV is denoted by angular brackets.

Rescaling and shifting $y\rightarrow(y-\Lambda^{N_c+2}v^{-1})v^{-1}$, the curve (2.2) becomes $f(y,v)=0$ where

$$f(y,v)\equiv y^3+y^2v\;p(v)+y\Lambda^{N_c+2}[q(v)-2p(v)] \\ +\Lambda^{2N_c+4}v^{-1}[p(v)-q(v)]. \tag{2.7}$$

Notice that $q(v)=p(-v)$ and then $v^{-1}[p(v)-q(v)]$ has no negative powers in v.

Although we do not know any field-theoretical method to obtain the curve for the case involving the antisymmetric tensor matter field, we can obtain rich information on the singular surface of the curve describing the Coulomb branch of $N=2$ SUSY Yang-Mills gauge theories by the method of "integrating in" [42,43]. We will here compute the singular surface of the proposed curve, where the discriminant vanishes. Since the antisymmetric tensor representation in the SU(3) gauge group is nothing but the antifundamental representation, we shall take the SU(4) gauge group as the simplest nontrivial case. Using Maple, in order to obtain the discriminant of the curve for SU(4) Yang-Mills theory with antisymmetric matter, we calculate

$$\prod_{i=1} f(y_i,v_i), \tag{2.8}$$

where (y_i,v_i) are solutions of simultaneous equations

$$\frac{\partial f}{\partial y}(y,v)=0, \tag{2.9}$$

$$\frac{\partial f}{\partial v}(y,v)=0. \tag{2.10}$$

To perform an explicit calculation, we take the $m=0$ case. The discriminant[1] is found to be

$$s_3^4\Delta^2\Delta_{\rm unphys}, \tag{2.11}$$

where

$$\begin{aligned}\Delta={}&191102976s_3^2\Lambda^{18}+(-1327104s_4s_2^4+5308416s_2^2s_4^2+110592s_2^6-8957952s_3^4-39813120s_2s_4s_3^2\\&-7077888s_4^3+9068544s_3^2s_2^3)\Lambda^{12}+(417792s_4^2s_2^5-1146880s_4^3s_2^3+139968s_3^6+245376s_3^4s_2^3+4096s_2^9\\&-488448s_2^4s_4s_3^2+59904s_3^2s_2^6+2211840s_4^3s_3^2+442368s_4^2s_2^2s_3^2-67584s_4s_2^7+1179648s_4^4s_2\\&+124416s_2s_4s_3^4)\Lambda^6-27648s_4^2s_3^4s_2^2-5632s_4^2s_3^2s_2^5-73728s_2s_4^4s_3^2+128s_3^2s_4s_2^7-256s_4^2s_2^8-16s_3^4s_2^6\\&+65536s_2^2s_4^5+38912s_4^3s_2^3s_3^2+7776s_3^6s_4s_2-24576s_2^4s_4^4-729s_3^8+2016s_3^4s_4s_2^4+13824s_4^3s_3^4\\&-65536s_4^6-216s_3^6s_2^3+4096s_4^3s_2^6,\end{aligned} \tag{2.12}$$

$$\Delta_{\rm unphys}=(-s_4^3+27\Lambda^6s_3^2). \tag{2.13}$$

Here s_i are the moduli parameters:

$$\langle\det(x-\Phi)\rangle=x^{N_c}+\sum_{i=2}^{N_c}x^{N_c-i}s_i. \tag{2.14}$$

The factor $\Delta_{\rm unphys}$ is believed to be unphysical [32]. On the other hand, the factor s_3^4 exhibits a singularity expected for the massless antisymmetric tensor matter field in the classical limit ($\Lambda\to 0$), as can be seen from the tree-level superpotential (2.4):

$$s_3^2=\left\langle\prod_{i>j}(\phi_i+\phi_j)\right\rangle. \tag{2.15}$$

It is interesting to observe that this singularity is identical to the classical limit ($\Lambda\to 0$) even though we are not restricted to the weak coupling case. In order to see the singularity associated with the massless gauge fields, we shall take the classical limit ($\Lambda\to 0$). Then the factor Δ becomes

$$\Delta\to\left\langle\prod_{i>j}(\phi_i-\phi_j)^4\right\rangle. \tag{2.16}$$

This is nothing but the classical singularity where the non-Abelian gauge symmetry is enhanced. We conclude that $s_3^2\Delta$ correctly reproduces the singularities in the classical limit.

III. ''INTEGRATING IN'' METHOD

In this section, we analyze the singular surface in the moduli space of the Coulomb branch by using the ''integrating in'' method in the field-theoretic framework. This method enables us to gain information on the singular surface in the Coulomb branch taking into account nonperturbative quantum effects.

The $N=2$ SUSY is broken to $N=1$ by adding a perturbation ΔW to the tree-level $N=2$ superpotential $W_{\rm tree}$ in Eq. (2.4):

$$\Delta W=\sum_{k=2}^{N_c}\frac{g_k}{k}{\rm Tr}(\Phi^k). \tag{3.1}$$

The classical VEV's of Φ are obtained from the classical equations of motion, $\partial(W_{\rm tree}+\Delta W)/\partial\Phi=0$, and similarly for $A^{ij},\tilde{A}_{ij}$. We are interested in the Coulomb branch, where $A^{ij}=\tilde{A}_{ij}=0$. After SU(N_c) rotations, the generic VEV can be reduced to $\Phi_{\rm cl}={\rm diag}(M,M,M_3,M_4,\ldots,M_{N_c})$, where $M=g_{N_c-1}/g_{N_c}$. In that case, the gauge group SU(N_c) is broken to $SU(2)\times U(1)^{N_c-2}$. The nonperturbative effects due to the gaugino condensation of SU(2) super Yang-Mills theory provides the additional superpotential

[1] Note that the antisymmetric representation of SU(4) is equivalent to the defining representation of SO(6), for which the Seiberg-Witten curve has been derived. It turns out that the discriminant of the Seiberg-Witten curve for the SO(6) with the defining representation contains the factor $\Delta_{\rm massive}$ which reduces to $s_3^2\Delta$ in Eq. (2.12) for the massless case.

$$W_d = \pm 2 g_{N_c} (\Lambda_{\rm FT}^{N_c+2} G)^{1/2}, \tag{3.2}$$

where

$$G = \prod_{p=3}^{N_c} (M_p + M + m), \tag{3.3}$$

and $\Lambda_{\rm FT}$ is the dynamical scale of the SU(N_c) gauge theory with antisymmetric matter in field theory. The $\Lambda_{\rm FT}$ must be proportional to Λ of the M theory brane configuration in the previous section:

$$\Lambda_{\rm FT} = c\Lambda, \quad c \in \mathbb{C}, \tag{3.4}$$

where c is a renormalization-scheme-dependent constant. Following Elitzur *et al.* [42], we obtain the low-energy effective superpotential for $N=1$ super Yang-Mills theory:

$$W_L = \Delta W[\Phi = \Phi_{\rm cl}(g_k)] + W_d + W_\Delta, \tag{3.5}$$

where W_Δ is a possible additional superpotential constrained only by holomorphy and symmetry [43].

The VEV of gauge invariants can be defined as

$$\langle u_k \rangle = \left\langle \frac{1}{k} {\rm Tr}(\Phi^k) \right\rangle, \tag{3.6}$$

which are obtained by differentiating W_L:

$$\langle u_k \rangle = \frac{\partial W_L}{\partial g_k}. \tag{3.7}$$

The Seiberg-Witten curve must be singular when $u_k = \langle u_k \rangle$. The VEV's are related to the moduli parameters s_i in Eq. (2.14) through the Newton formula

$$k s_k = -\sum_{j=1}^{k} j s_{k-j} \langle u_j \rangle, \tag{3.8}$$

with $s_0 = 1$, $s_1 = 0$.

As a simplest explicit example, we consider the SU(4) case. Then, the VEV's and the gaugino condensation are given in terms of coupling parameters in the superpotential as

$$M = z_3, \tag{3.9}$$

$$M_3 = -z_3 + \sqrt{-z_3^2 - z_2}, \tag{3.10}$$

$$M_4 = -z_3 - \sqrt{-z_3^2 - z_2}, \tag{3.11}$$

$$G = m^2 + z_3^2 + z_2, \tag{3.12}$$

where z_3 and z_2 are complex parameters:

$$z_3 \equiv \frac{g_3}{g_4}, \quad z_2 \equiv \frac{g_2}{g_4}. \tag{3.13}$$

In the remainder of the paper, we will explore the singular surface by using the ''integrating in'' method, in order to compare it with curve obtained from the M theory five-brane. First, we assume $W_\Delta = 0$ in this section. Then we find the VEV including quantum effects using Eq. (3.7) as

$$\begin{aligned} \langle u_2 \rangle &= z_3^2 - z_2 \pm \Lambda_{\rm FT}^3 G^{-1/2}, \\ \langle u_3 \rangle &= 2 z_3^3 + 2 z_3 z_2 \pm 2 z_3 \Lambda_{\rm FT}^3 G^{-1/2}, \\ \langle u_4 \rangle &= -\frac{3}{2} z_3^4 - 2 z_3^2 z_2 + \frac{1}{2} z_2^2 \\ &\quad \pm (2m^2 + z_2) \Lambda_{\rm FT}^3 G^{-1/2}. \end{aligned} \tag{3.14}$$

These relations define a codimension-1 surface in moduli space. It should correspond to the singular surface of the proposed curve (2.7) for SU(4) with antisymmetric tensor matter, namely, the vanishing discriminant of the curve. We will find, however, that the discriminant of the curve (2.7) does not vanish on $u_k = \langle u_k \rangle$ in Eq. (3.14) for the case $m = 0$.

We shall now test if the discriminant vanishes for any values of coupling parameters z_3 and z_2 by choosing an appropriate value for the renormalization-scheme-dependent factor c in Eq. (3.4). We find that the factor Δ in the discriminant (2.11), for instance, becomes, on the codimension-1 surface (3.14) for $m=0$,

$$\begin{aligned} \Delta(u_k = \langle u_k \rangle) &= -1024 (c^6 - 4)(z_3^2 + z_2)^3 (5 z_3^2 + z_2)^6 \Lambda^6 \\ &\quad + [(811 c^6 - 3744) z_3^6 + (117 c^6 - 768) z_3^4 z_2 \\ &\quad + (-327 c^6 + 1248) z_3^2 z_2^2 + (47 c^6 - 192) z_2^3] \\ &\quad \times 512 (z_3^2 + z_2)^{3/2} (5 z_3^2 + z_2)^3 c^3 \Lambda^9 + \mathcal{O}(\Lambda^{12}) \\ &\not\approx 0. \end{aligned} \tag{3.15}$$

To be more precise, there is no complex number c satisfying $\Delta(u_k = \langle u_k \rangle) \equiv 0$ for any values of z_3 and z_2. Therefore the discriminant of the curve (2.7) does not vanish on the codimension-1 surface (3.14) obtained by assuming $W_\Delta = 0$ in the ''integrating in'' method. Although we have no rigorous means to test the curve (2.2) obtained in M theory for general N_c, we are confident that the curve is correct at least for $N_c = 4$, since we have checked that the discriminant agrees with that of SO(6) with the defining representation. Therefore we conclude that the assumption $W_\Delta = 0$ in the case of SU(4) theory with antisymmetric tensor matter leads us to inconsistent results and that the assumption $W_\Delta = 0$ is not correct.

IV. NONZERO W_Δ

In the previous section, we found that the M theory curve (2.7) is inconsistent with the codimension-1 surface obtained as a candidate for the singular surface in moduli space assuming $W_\Delta = 0$ in the ''integrating in'' method. In this section, we discuss the possibility of nonzero W_Δ instead.

We first note that, in the classical limit ($\Lambda \to 0$), the discriminant of the M theory curve (2.7) vanishes on the codimension-1 surface (3.14) obtained by assuming $W_\Delta = 0$ in the ''integrating in'' method. Equation (3.15) shows that the discriminant of the curve vanishes on the codimension-1 surface in the leading order of Λ, i.e., up to order Λ^6, provided $c^6 = 4$:

$$\Lambda_{\rm FT}^6 = 4\Lambda^6, \quad W_d = \pm 4 g_4 \Lambda^3 G^{1/2}. \tag{4.1}$$

Now we wish to explore to higher orders of Λ whether we can find a nontrivial W_Δ which provides a singular surface consistent with the vanishing discriminant of the curve. Since the right-hand side of Eq. (3.15) consists of terms with integer powers of Λ^3, we need to introduce terms with Λ^{3n} $n \in \mathbb{N}$ only:

$$W_\Delta = \sum_{k=2}^{\infty} C_k (\Lambda^3)^k. \tag{4.2}$$

Since the Λ^3 term is given by W_d, we assume $k \geqslant 2$.

The additional superpotential W_Δ must satisfy the following conditions [43]:

$$\begin{aligned} &W_\Delta \to 0 \quad \text{as } g_2 \to \infty \text{ with } g_4 \Lambda^3 G^{1/2} \text{ fixed,} \\ &W_\Delta \to 0 \quad \text{as } \Lambda \to 0, \end{aligned} \tag{4.3}$$

and carry charge $(2,2)$ under $\mathrm{U}(1)_R \times \mathrm{U}(1)_J$.

We list below the charge and mass dimension of the parameters:

	$\mathrm{U}(1)_R$	$\mathrm{U}(1)_J$	Dimension
g_2	-2	2	1
g_3	-4	2	0
g_4	-6	2	-1
z_3	2	0	1
z_2	4	0	2
Λ	2	0	1
G	4	0	2
m	2	0	1

(4.4)

From this table, we see that W_Δ can be represented as

$$W_\Delta = g_4 \sum_{k=2}^{\infty} f_k(z_3, z_2) \Lambda^{3k}, \tag{4.5}$$

where $f_k(z_3, z_2)$ is any function which carries the charge $(4-6k, 0)$. Note that we are discussing SU(4) Yang-Mills theory with a massless ($m=0$) antisymmetric tensor.

Now let us determine the lowest term of W_Δ in order to make the singular surface consistent with the discriminant of the M theory curve. If $W_\Delta = W_2 \equiv g_4 f_2(z_3, z_2) \Lambda^6$, then

$$\begin{aligned} \langle u_2 \rangle &= z_3^2 - z_2 \pm 2\Lambda^3 G^{-1/2} + \frac{1}{g_4} \frac{\partial W_2}{\partial z_2}, \\ \langle u_3 \rangle &= 2z_3^3 + 2z_3 z_2 \pm 4 z_3 \Lambda^3 G^{-1/2} + \frac{1}{g_4} \frac{\partial W_2}{\partial z_3}, \\ \langle u_4 \rangle &= -\frac{3}{2} z_3^4 - 2z_3^2 z_2 + \frac{1}{2} z_2^2 \pm 2 z_2 \Lambda^3 G^{-1/2} \\ &\quad - \frac{z_2}{g_4} \frac{\partial W_2}{\partial z_2} - \frac{z_3}{g_4} \frac{\partial W_2}{\partial z_3} + \frac{W_2}{g_4}. \end{aligned} \tag{4.6}$$

From this, we find

$$\begin{aligned} \Delta(u_k = \langle u_k \rangle) = &\mp 2048 (z_3^2 + z_2)^{3/2} (5 z_3^2 + z_2)^6 [f_2(z_3, z_2) \\ &\times (z_3^2 + z_2) - 2] \Lambda^9 + \mathcal{O}(\Lambda^{12}). \end{aligned} \tag{4.7}$$

Thus, in order for Δ to vanish up to Λ^9, W_2 must take the form

$$W_2 = 2 g_4 (z_3^2 + z_2)^{-1} \Lambda^6 = 2 g_4 G^{-1} \Lambda^6. \tag{4.8}$$

This W_2, Eq. (4.8), satisfies the conditions (4.3) and carries the charge (2,2).

We can determine the next term of W_Δ in a similar way and find

$$\begin{aligned} W_d + W_\Delta = &\pm 4 g_4 \Lambda^3 G^{1/2} + 2 g_4 \Lambda^6 G^{-1} \mp 2 g_4 \Lambda^9 G^{-5/2} \\ &+ \mathcal{O}(\Lambda^{12}). \end{aligned} \tag{4.9}$$

Inspired by the result (4.9), we restrict the form of W_Δ in the following way:

$$W_d + W_\Delta = g_4 \sum_{k=1}^{\infty} h_k G^2 (\Lambda^3 G^{-3/2})^k, \quad \text{where } h_k \in \mathbb{C}. \tag{4.10}$$

By requiring Δ to vanish, we work out h_k up to h_8,

$$\begin{aligned} W_d + W_\Delta = &\pm 4 g_4 \Lambda^3 G^{1/2} + 2 g_4 \Lambda^6 G^{-1} \mp 2 g_4 \Lambda^9 G^{-5/2} \\ &+ 4 g_4 \Lambda^{12} G^{-4} \mp \frac{21}{2} g_4 \Lambda^{15} G^{-11/2} + 32 g_4 \Lambda^{18} G^{-7} \\ &\mp \frac{429}{4} g_4 \Lambda^{21} G^{-17/2} + 384 g_4 \Lambda^{24} G^{-10} + \mathcal{O}(\Lambda^{27}), \end{aligned} \tag{4.11}$$

and we find that the discriminant vanishes up to the order Λ^{27}:

$$\Delta(u_k = \langle u_k \rangle) = \mathcal{O}(\Lambda^{30}). \tag{4.12}$$

Although we have only determined W_Δ up to this order due to the increasing complexity of computation, we believe that the higher powers of Λ can be worked out with more effort and that the form (4.10) will come out.

V. DISCUSSION

In this paper, we studied the singular surface of the moduli space of $N=2$ SUSY SU(N_c) gauge theory with antisymmetric tensor matter from two points of view. One is to use a configuration of a single M theory five-brane and the other is based on the ''integrating in'' method. It was discussed that the consistency between the two results requires $W_\Delta \neq 0$, and the explicit form of it was worked out for $N_c=4$. It is interesting that W_Δ consists of terms with integer powers of $\pm\Lambda^3$ corresponding to two vacua of SU(2) gaugino condensation. Using the dynamical scale $\Lambda_{SU(2)}$ of unbroken SU(2) SUSY Yang-Mills theory instead of Λ, the nonperturbative superpotential (4.10) is rewritten as

$$W_d + W_\Delta = g_4 \sum_{k=1} h_k G^2 \left(\pm \frac{\Lambda_{SU(2)}^3}{2 g_4 G^2} \right)^k, \tag{5.1}$$

where the scale matching condition $\Lambda_{SU(2)}^3 = 2 g_4 G^{1/2} \Lambda^3$ for $N_c=4$ is used. This fact makes us suspect that the physical origin of W_Δ might be understood as the gaugino condensation of SU(2) SUSY Yang-Mills theory.

In order to break $N=2$ SUSY to $N=1$ SUSY, we added a perturbation (3.1) to the tree-level $N=2$ superpotential $W_{\rm tree}$ in the ''integrating in'' methods. Instead of this perturbation, we can consider another perturbation

$$\Delta W = \frac{g_2}{2} \mathrm{Tr}(\Phi^2) + \frac{g_3}{3} \mathrm{Tr}(\Phi^3) + g_4 \left[\frac{1}{4} \mathrm{Tr}(\Phi^4) - \alpha \left(\frac{1}{2} \mathrm{Tr}(\Phi^2) \right)^2 \right], \quad \alpha \in \mathbb{C}, \tag{5.2}$$

to the tree-level $N=2$ superpotential $W_{\rm tree}$. It turns out that for $\alpha \neq 1/2$ there exist classical vacua where the gauge group is broken to $SU(2)\times U(1)^2$ [48]. One can then calculate W_Δ for generic α by requiring that $\Delta=0$ in the $u_k = \langle u_k \rangle$ surface. One of the most interesting results is that W_Δ vanishes for $\alpha = 1/4$ [51] while $W_\Delta \neq 0$ for any other value of α. This is presumably related to the fact that the antisymmetric representation of SU(4) is equivalent to the defining representation of SO(6), for which the assumption $W_\Delta=0$ is found to be valid.

It is interesting to note that the ''integrating in'' methods have been applied under the assumption $W_\Delta=0$ [42–48], which provides consistent results in the Seiberg-Witten curves that are hyperelliptic. It has been known that the nonhyperelliptic Seiberg-Witten curves for the exceptional group cases are derived using the assumption $W_\Delta=0$ [45,48]. Contrary to these results, we have found that the assumption $W_\Delta=0$ is inconsistent in the case of $N=2$ SUSY SU(N_c) gauge theory with antisymmetric tensor matter, whose Seiberg-Witten curve is not of hyperelliptic type [32]. As another example of nontrivial W_Δ, we have studied also $N=2$ SU(4) theory with a symmetric tensor whose Seiberg-Witten curve is proposed in Ref. [32]. In this case, we find that there is no complex number α in Eq. (5.2) to make W_Δ vanish. On the other hand, we also find that for fundamental matter, W_Δ vanishes for any value of α ($\alpha \neq 1/2$).

Higher values of N_c may be dealt with by a similar method with more computational effort. We expect that $W_\Delta \neq 0$ for SU(N_c) with an antisymmetric or symmetric representation for higher values of N_c also.

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